Progress Report 84 December 1959

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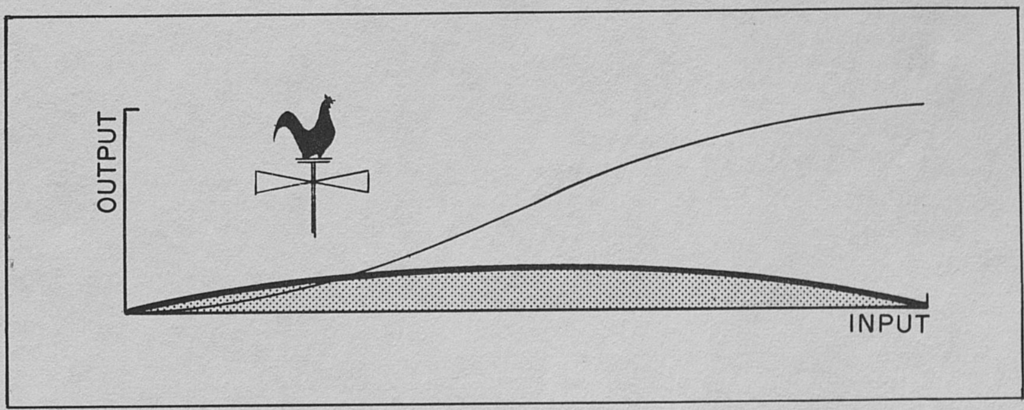

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CHANGES IN THE WEATHER AND PRODUCTION FUNCTIONS FOR A SAMPLE OF WEST KENTUCKY FARMS

By A. N. Halter and G. L. Bradford*

Farms in three TVA counties of Western Kentucky that were surveyed in 1953 by Jensen and Sundquist[1] were revisited in 1957 to ascertain the changes that occurred in response to input and output prices. Specifically we wanted to determine: (1) whether the production function for the sample of farms had changed, (2) the extent of changes in input combinations and levels of income that occurred, and (3) the adjustment of inputs towards their least cost combination in response to changes in prices that occurred over the four-year period. In addition, certain measurements of managerial capacity were made. Mana... factors that were thought to be associated with the productivity of the variable inputs and the adjustments of inputs toward their least cost combination were measured. Although this latter phase including number three above is being studied,[2] this progress report presents the important results of one and two.

This report discusses: first, changes in the sample; second, estimation of the production function including (a) specification of inputs and income and (b) correction of data for changes in price level and differences in weather; and third, the major results including (a) testing for a change in the production function and (b) marginal productivities of inputs from the uncorrected and weather-adjusted functions.

CHANGES IN THE SAMPLE

The Jensen and Sundquist[3] survey provided the basic data for 1952.[4] The sampled area was restricted to upland soils consisting of approximately 85 percent Grenada and 15 percent Calloway silt loam soils of Calloway, Graves, and

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*Parts of this report came from G. L. Bradford, Determination of Changes in Technology on a Sample of Western Kentucky Farms (Unpublished M.S. Thesis, University of Kentucky, 1959).

[1]H. R. Jensen and W. B. Sundquist, Resource Productivity and Income for a Sample of West Kentucky Farms, Ky. Agr. Exp. Sta. Bul. 630 (Lexington June, 1954).

[2]This phase of the research is being done by D. G. Paris for his Ph.D. dissertation. Appreciation is due him for aid and suggestions in preparation of this report.

[3]Jensen and Sundquist, op. cit, pp. 7-8.

[4]Basic data included the quantity figures and prices for the input and output items. Other information was obtained from U. S. Department of Agriculture publications, TVA publications, and Kentucky Agricultural Experiment Station sources.

Marshall counties. Uniformity in output combinations is implied by the sample being from a contiguous homogeneous area. A block random sampling method was used, and data were collected from 144 farms.

Whenever possible, the same farmers were revisited in 1957, and basic data were collected for the same input and output items for 1956. Thus, the sampling considerations apply to both years. If a farmer had moved to another farm in the area specified above, he was interviewed again. However, it was found that 27 of the 144 farmers had either died, moved out of the area, or declined to be interviewed. In these cases it was necessary to replace them with units of as similar characteristics as possible.

Any great error in replacing these 27 units would have distorted the representativeness of the sample. Since predictions were to be made using the 1952 data as a basis, it was important to notice that the sample of 144 farms was representative for both years. The 117 farms visited in 1952 and 1956 had an average gross income of $4,437 and $6,562 respectively.[5] The 27 farms sampled only in 1956 had an average of $7,196. The difference in gross income between the two years for the 117 farms was $2,125 and for the 27 farms is $2,453. The change in gross income between the two years was not significantly different when the "t" test was applied to the two sets of farms. With this information it was concluded that the sample was not materially changed by the new farms surveyed in 1957.

ESTIMATION OF THE PRODUCTION FUNCTION

The estimation of the production function or the input-output relationship that existed on this sample of farms for 1952 was carried out in a previous productivity study.[6] However, since a number of new techniques were used in attempting to reduce the high intercorrelations between inputs that were present in the Jensen and Sundquist study, the function estimation was re-examined, and therefore details of the analysis are presented here. Also, since the 1952 function coefficients were going to be used to test if the function had changed, i.e., if technology had changed by 1956, a number of adjustments for price level had to be made in the data. These are also presented in this section. The general nature of the production function is $Y = f(X_1, X_2, X_3, X_4, W, U)$ where:

Y denotes gross income,

X_1 denotes acres of land and associated inputs,

X_2 denotes days of labor,

X_3 denotes services from forage machinery, forage and nonforage consuming livestock, and livestock buildings.

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[5]The 1952 items were enumerated in 1956 dollars as will be explained later.

[6]Jensen and Sundquist, *op. cit.*

X_4 denotes purchased inputs,

W denotes weather, and

U denotes unexplained residuals due to omitting variables from the equation.

Specification of Input Categories

All the items of gross income (Y) and input categories X_3 and X_4 were enumerated in dollars.[7] Thus, the observations (for each farm) for these three variables were determined by adding the value of each item in dollars. Quantity figures for some of the items were determined in the survey (for 1952), while other items were listed in total dollars sold or paid.[8] The procedure was to enumerate items in 1952 dollars and then to convert to 1956 dollars by a price index or price relative.[9] The corresponding items for 1956 were, of course, enumerated in 1956 dollars.

The following formula was used to adjust the items

$$X_{k_{(o)}} \cdot I_{k_{(n)}} = X_{k_{(n)}}$$ where,

X signifies the input or output item, I the index or price relative, and the subscripts k, o, and n respectively denote the kth item, the base year (1952), and the nonbase year (1956). The price relative (I) was computed by dividing the 1956 price by the 1952 price and multiplying by 100. Each item of the three categories (Y, X_3 and X_4) was adjusted for each farm for 1952. This amounts to the same as adjusting the entire category by a weighted aggregative index of the form

$$I = \frac{q_{(n)} \; X_{(n)}}{q_{(o)} \; X_{(o)}}$$ where,

q denotes the price, X denotes the input or output item, and the subscripts o and n denoting the base year and the nonbase year. In general, the prices of input items were higher in 1956 than in 1952, and the prices of gross income items had declined. This is what would have been expected since the overall index of prices paid by farmers has been increasing, and the prices received fell during the period. Thus, the importance of stating the 1952 items in 1956 dollars cannot be overemphasized since the effect of a change in technology (if any) would have been obscured by the price changes.

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[7]See Appendix A for a discussion of how the items were first enumerated and a list of the prices used if the item was not already scheduled in dollars.

[8]Had quantity figures been available for all items, then the items could have been evaluated in 1956 dollars simply by multiplying the quantity by the 1956 price.

[9]See Appendix A for a list of indices.

Land and Associated Inputs

In general it is desirable to group input items that are complements or substitutes together. However, in actual practice many of the input items considered are neither perfect complements nor perfect substitutes or near-substitutes. The input categories were specified with the idea that they were not complements or substitutes, i.e., the correlation between categories was assumed to be fairly low.[10]

Specifying X_1

Most of the items in this category were assumed to be used by farmers in more or less fixed proportions with land (operating acres). For example, machinery depreciation increases as operating acres increase and decreases as operating acres decrease. These proportions were not necessarily constant, and the exact relationship was not known when the items were grouped into the category. However, preliminary analysis and past experience gave good indication that the items were complements or near-complements.[11] Thus, the input could be specified as a single item or as an index or "set number" representing all the items. For example, gross income is a function of operating acres and machinery depreciation, i.e., $Y = f(L, D)$, and if $D = g(L)$ then operating acres and machinery depreciation may be specified as one variable, thus, $Y = h(L)$. The problem is the same conceptually when more than two variables are considered.

Using the above logic as a guide, eight items of the X_1 category were correlated with an index of gross income which was to reflect the level of use of this input category.[12] A linear function of the form was used

$$Y = a + b_1V_1 + \dots + b_8V_8 + U$$ where,

Y denotes the index of gross income[13] and the following items were represented by the V_i variables: (1) operating acres which included all farm acreage except woodland, wasteland, and unused lots; (2) tobacco acreage which included burley,

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[10]See Appendix B for a list of intercorrelation figures for the current study and for the Jensen and Sundquist study.

[11]A similar problem will occur when a majority of the farm firms are combining the inputs in least cost combinations. High intercorrelation will exist when a majority of the sampled farm firms are producing with least cost combinations. This high correlation occurs because the scatter of points will lie along the scale line or expansion path, e.g., when one input is increased the other is increased in direct proportion. This phenomenon may make it impossible to derive parameters for the input categories since there will be no scatter of points to which to fit the function.

[12]Any other variable highly correlated with these items could have been used to construct the index and reflect the level of combined usage.

[13]This index was computed for each observation by dividing the dollars of gross income for each farm by the arithmetic mean of all farms and multiplying the result by 100.

dark-fired, and dark air-cured tobaccos; (3) machinery depreciation which included tractors, trucks, cornpickers, grain drills, and combines; (4) machinery repairs, (5) tobacco barn area; (6) fence depreciation; (7) fuel expense; and (8) crop seed expense.[14] This procedure was followed for both 1952 and 1956, since the 1956 inputs were also needed in testing the hypothesis that the technology had remained constant.

Coefficients derived for each of these items are presented in Table 1. The "set number" was estimated for each observation by evaluating the function using the regression coefficients and the input observations for that farm. The range of the "set numbers" for 1952 was 14 to 442 and from 1956 was 16 to 402.

Table 1. - Coefficients of proportionality for items of variable land and associated inputs.[a]

Item	Coefficient
V_1 - Operating acres	.17397
V_2 - Tobacco acreage	6.64
V_3 - Machinery depreciation[b]	.06153
V_4 - Machinery repairs	.08171
V_5 - Tobacco barn area	.01376
V_6 - Fence depreciation	.00381
V_7 - Fuel expense	.01916
V_8 - Crop seed expense	.10156

[a]Based upon 1952 data only.

[b]Includes only tractors, trucks, cornpickers, grain drills, and combines.

Labor, Livestock-forage Input, and Purchased Inputs

The labor variable (category X_2) was specified in total days per farm including hired labor, family labor, and the manager's labor. The observations were determined for each year by determining the number of days of labor on each farm. No adjustment was necessary on the 1952 data since it was enumerated in physical units.

The variable input X_3 denotes services from forage machinery, forage consuming livestock, grain consuming livestock, and livestock buildings. Machinery items included hay balers, forage harvesters, trailers and wagons, ensilage cutters, feed grinders, mowing machines, milkers, and milk coolers. Livestock

[14]The items which were in 1952 dollars were first adjusted to 1956 dollars.

included dairy and beef breeding animals, sheep breeding animals, swine breeding animals, and beginning inventory value of home-produced feeder animals. Buildings included silos, dairy barns, and cattle sheds.

The variable input X_4 denotes purchased inputs which included purchased feeder livestock; fertilizer and lime; electricity, telephone, and automobile expense (farm share); spray materials; seed treatment; breeding fees; baby chicks; custom hired machinery services including corn picking, bulldozing, and trucking; and sawdust and slab wood.

Specifying the Weather Variable

The general practice in productivity studies has been to disregard the effects of the exogenous variable weather. However, in this study this variable cannot be ignored since the results depend directly upon taking account of the major changes or differences between 1952 and 1956 in the variables influencing production. Generally speaking, 1952 was a dry, hot year while 1956 was much more seasonable. More specifically the weather variable may be considered or specified in the following manner:

1. Consider again the function for 1952 in the general form

$$Y = f(X_1, X_2, X_3, X_4, W, U).$$

2. The equality still holds when manipulated, thus

$$\frac{Y}{W} = \frac{f(X_1, X_2, X_3, X_4, W, U)}{W}$$

$$Y' = f(X_1, X_2, X_3, X_4, U).$$

The weather variable was considered by this method, i.e., an adjustment factor for weather was computed for 1956 (1952 being 100) and multiplied by the 1952 gross income, or division by the reciprocal of the factor as in 2. above.

Derivation of the Weather Adjustment Factor

The steps in deriving the factor were: (1) correlation was established between weather and crop yields, and (2) farms with different crops and acreages were weighed such that the separate effect of differences in weather could be determined for each farm. Specifically, crop yield data for corn, wheat, and hay were regressed upon weather data in an attempt to specify correlation between the two. The crop yield data were taken from the Mayfield Soil Experiment Field for the years 1927 through 1954, excluding 1943 and 1944.[15] The weather data

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[15]See P. E. Karraker and Harold F. Miller, A Summary of Kentucky Soil Fertility Experiments, Ky. Agr. Exp. Sta. Bul. 663 (Lexington, June, 1958).

used were the number of drought days for April through October for the corresponding years.

Computation of Drought Days

Drought days, for April through October, from the Paducah weather station for the years 1927 through 1954, excluding 1943 and 1944, were used as the independent variables in the correlation of crop yield data.[16] Rainfall and temperature data alone ignore certain plant and soil characteristics which in a large part determine crop yields. However, in the number of drought days several variables directly or indirectly make up the weather variable. The number of drought days is an index determined by several relevant climatic and agronomic factors which bring about drought conditions.

The number of drought days for each month are computed from rainfall and evapotranspiration data by a moisture-balance method. The daily evapotranspiration was calculated by the Penman formula by Knetsch and Smallshaw.[17] The different water-holding capacities and the number of drought days had to be considered for each level, since the exact water-holding capacity of the soil in the area is also a variable. For each of five levels of moisture[18] the rainfall is added for each day it occurs during the month, and the calculated inches of evapotranspiration are subtracted from the available amount of soil moisture for each day. For example, if the available soil moisture is .31 inches for the 1-inch moisture level and .55 inches for the 3-inch level on the first day of the month, and the evaptranspiration was .17 inches per day, then within two days of moisture would be exhausted for the 1-inch level, and within four days the moisture would be exhausted for the 3-inch level. Every day after the moisture has been depleted is a drought day until it rains. Rainfall, when it occurs, is added to the available soil moisture. There will naturally be more drought days for the 1-inch level than for the higher levels, since the base amount of available soil moisture is less.

Regression of Yields on Drought Days

Yields of corn, wheat, and hay were regressed upon the number of drought days and time (to establish trend, if existing) for the years 1927 through 1954, excluding 1943 and 1944. The number of drought days for each month, April through October, were the independent variables in this regression.[19] The corn

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[16] For a tabulation of data for the Paducah station see Jack L. Knetsch and James Smallshaw, The Occurrence of Drought in the Tennessee Valley, Report T 58-2 AE, (Knoxville: Tennessee Valley Authority, June, 1958), p. 47.

[17] *Ibid.*, pp. 5-7.

[18] The quantity of moisture available to crops was determined by multiplying the available moisture-holding capacity of the soil, in inches per foot, by the effective rooting depth of a crop. Thus, a soil is said to have 4 inches of available soil moisture or 6 inches, or 7 inches. The five levels used here were 1, 2, 3, 4, and 6 inches.

[19] Knetsch and Smallshaw, *op, cit.*, p. 47

wheat, and hay yield figures were obtained from plots 4, 5, and 6 of the Mayfield Soil Experiment Field.[20] This regression was performed for all five moisture levels, regressing the yield of each plot and the average of the three plots upon the drought days and upon time. Drought days for August through October were lagged for hay. For example, drought days for July through October of 1930 and drought days for April through June of 1931 were paired with 1931 yield data. Thus, in the first regression 25 observations since 1943 and 1944 were omitted and the results were lagged leaving out 1927. Specifically these functions may be symbolized:

$$1.\quad C = a + b_1Z_{1(t)} + b_2Z_{2(t)} + b_3Z_{3(t)} + b_4Z_{4(t)} + b_5Z_{5(t)} + b_6Z_{6(t)} + b_7Z_{7(t)} + b_8t$$

$$2.\quad W = a + b_1Z_{1(t)} + b_2Z_{2(t)} + b_3Z_{3(t)} + b_4Z_{4(t)} + b_5Z_{5(t-1)} + b_6Z_{6(t-1)} + b_7Z_{7(t-1)} + b_8t$$

$$3.\quad H = a + b_1Z_{1(t)} + b_2Z_{2(t)} \pm b_3Z_{3(t)} + b_4Z_{4(t-1)} + b_5Z_{5(t-1)} + b_6Z_{6(t-1)} + b_7Z_{7(t-1)} + b_8t$$

where,

C denotes corn yield in bushels, W denotes wheat yield in bushels, H denotes hay yield in pounds, Z_1 through Z_7 denote drought days of April through October respectively, and t denotes the current year with t-1 denoting the previous year. The parameters of these equations were calculated for each of the five moisture levels and for each of the yields of the three plots (4, 5, 6) and their average.

The regression coefficient b_8 was significant for corn and wheat; hence, the effect of time was removed from the yield data;[21] hay showed no trend. The deviations from the trend line for corn and wheat were converted to indices. Yields lying above the trend line were entered as an index above 100 and vice versa. Then, the corn and wheat indices of yield and the hay yields were again regressed upon the drought days data; only the average of the yields from the three plots were used in this case as the dependent variable.[22] After this second regression, one level

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[20] Karraker and Miller, op, cit., pp 19-21. Also from records of the Agronomy Department, Kentucky Agricultural Experiment Station. These plots had relatively similar treatments.

[21] The moisture levels which had the highest R^2 value were selected as the function to be used for each crop. Then, for this moisture level only the regression coefficients which were significant were used.

[22] The average yield of the three plots was used since the R^2 value was higher than for the individual plots.

of moisture and the appropriate months were chosen for each of the crops in order that an index or crop yield could be predicted for each year (1952 and 1956). For corn, moisture level three (3 inches) had the highest R^2 value, and the regression coefficients for the months of July and August were significant. For wheat, moisture level three had the highest R^2 value, and coefficients for July for the present year and August of the previous year were significant. For hay, moisture level one was used (highest R^2), with coefficients of July of the present year and October of the previous year. These variables were used to calculate the following values: (1) corn index 98.8; in 1952 and 109.9 in 1956, (2) wheat index 108.7 in 1952 and 100.0 in 1956 and (3) hay 4,618.8 pounds per acre in 1952 and 8,139.5 in 1956. The adjustment factors were obtained by expressing the 1956 index (yield in the case of hay) as a percentage of the 1952 index (yield). These factors were 110 for corn, 92 for wheat, and 176 for hay.

Adjustment Factor for Each Farm. - The final step in the adjustment technique consisted of determining weights to give to each of the adjustment factors for each farm. The method used was to determine for each of the 144 farms the percentage of the total operating acres devoted to (1) corn, including popcorn and field corn, (2) small grains, since they are all similar to wheat, and (3) pasture and hay acreage. On most farms these three categories included over 90 percent of the total operating acres. Each of these percentages was respectively multiplied by the adjustment factors determined above and the results added together, giving a weighted adjustment factor for each farm. This was multiplied by the 1952 gross income, and the resultant product was the 1952 gross income in terms of 1956 weather. It is interesting to note that for almost all of the 144 farms the income was adjusted upward; most of the adjustment indices were between 140 and 150. This is quite consistent with the idea that 1952 was a dry, hot year while 1956 was much more seasonable. Hence, regardless of the crops grown on the various farms, income was adjusted upward.

TESTING FOR A CHANGE IN THE PRODUCTION FUNCTION

The 1952 function in general notation now appears as follows:

$Y' = f(X_1, X_2, X_3, X_4, U)$ where the variable inputs represent the items defined above and Y' represents the adjusted gross income for 1952. Least-squares regression was used to derive the parameters of the Cobb-Douglas and the transcendental equations in their logarithmic forms. This method is standard procedure and need not be discussed here.

The Cobb-Douglas equation which was fitted to the adjusted 1952 data[23] allows for either increasing, constant, or decreasing marginal returns throughout the production relationship. The transcendental equation, also fitted to the data, can

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[23]The form of the equation with U = residuals is $Y = aX_1^{b_1}X_2^{b_2}X_3^{b_3}X_4^{b_4}U$ and in logarithms is $\ln Y = \ln a + b_1 \ln X_1 + b_2 \ln X_2 + b_3 \ln X_3 + b_4 \ln X_4 + \ln U$. The residual is assumed to be due to errors in the equation.

exhibit nonconstant elasticity, i.e., increasing, decreasing, and negative marginal returns, singularly, in pairs, or all three simultaneously.[24] Thus, if the data clearly indicates three stages, then the parameters when fitted will show three stages. Regression coefficients and related statistics are shown for the transcendental and the Cobb-Douglas equations in Tables 2 and 3. The coefficient of determination (R^2) was .70 for the Cobb-Douglas and .72 for the transcendental function. Thus, approximately 30 percent of the variation in gross income was left unexplained by the five independent variables (including weather) of the 1952 function.

The major hypothesis of this phase of the research was that the production function had not changed between 1952 and 1956. To test this hypothesis, the predicted gross income for 1956 was compared to the actual 1956 gross income. The predicted gross income was calculated by evaluating Y (i.e., 1952 parameters) with the 1956 input observations. This is made explicit in the following formula for the logarithmic form of the transcendental equation:

$$\ln Y' = \ln c_{(o)} + a_{1(o)} \ln X_{1(n)} + b_{1(o)} X_{1(n)} + a_{2(o)} \ln X_{2(n)} + b_{2(o)} X_{2(n)} + a_{3(o)} \ln X_{3(n)} + b_{3(o)} X_{3(n)} + a_{4(o)} \ln X_{4(n)} + b_{4(o)} X_{4(n)}$$

where the subscripts 1 to 4 indicate the input category, o denotes the base year (1952) and n denotes the nonbase year (1956). The predictions were made by the same method for the Cobb-Douglas equation.

The predicted and the actual gross income for 1956 were compared. The magnitude of this difference was determined by a chi-square statistic. The chi-square test provided a means of discovering if the deviation was larger than what would be expected to be due to chance alone. This test was performed by the following formula:

$$\text{SUM } \frac{(a - e)^2}{e}$$

where,

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[24] The form of this equation with U = residuals is

$$Y = cX_1^{a_1} e^{b_1 X_1} X_2^{a_2} e^{b_2 X_2} X_3^{a_3} e^{b_3 X_3} X_4^{a_4} e^{b_4 X_4} U$$ and in logarithms is

$$\ln Y = \ln c + a_1 \ln X_1 + b_1 X_1 + a_2 \ln X_2 + b_2 X_2 + a_3 \ln X_3 + b_3 X_3 + a_4 \ln X_4 + b_4 X_4 + \ln U.$$

For a detailed discussion of the mathematical properties of this function see A.N. Halter, H. O. Carter, and J. G. Hocking, "A Note on the Transcendental Production Function," Journal of Farm Economics, XXXIX (November, 1957), pp. 966-974.

TABLE 2. - Regression coefficients and the standard errors of the mean marginal productivities for the 1952 transcendental function adjusted for weather.

Input	Category	Regression Coefficients[a]	Mean Marginal Productivity	Standard Error of Mean Marginal Productivity[b]
$\ln X_1$		.43938		
X_1	(index)	.00091	33.73	.92
$\ln X_2$		.23018		
X_2	(days)	-.00007	3.91	.14
$\ln X_3$		.03232		
X_3	(dollars)	.00011	1.13	.09
$\ln X_4$		.19159		
X_4	(dollars)	.00002	1.05	.05
c	(constant)	39.73760		

[a]The function coefficient or the "returns to scale" is 1.04 at the mean level of inputs.

[b]The definition of the variance of marginal productivities is

$$\sum_{i=1}^{N} \frac{(MP_i - \overline{MP})^2}{d.f.}$$ where,

MP is the computed marginal productivity for the observed level of input, $\overline{MP}$ is the marginal productivity computed at the mean level of inputs, and d.f. stands for the appropriate degrees of freedom. For this function the arithmetic means of the inputs were used. N = 135. The standard error is the square root of the variance divided by the square root of N.

TABLE 3. - Regression coefficients, their standard error, and standard error of the mean marginal productivities for the 1952 Cobb-Douglas function adjusted for weather.

Input	Category	Regression Coefficients[a]	Standard Error of Regression Coefficients	Mean Marginal Productivity	Standard Error of Mean Marginal Productivity[b]
X_1	(index)	.51226	.02345	31.79	.71
X_2	(days)	.24420	.02236	4.27	.16
X_3	(dollars)	.05178	.00632	1.25	.12
X_4	(dollars)	.24321	.01788	1.39	.06
a	(constant)	19.34317			

[a]The function coefficient or the "returns to scale" is 1.05

[b]For this function the geometric means were used in the definition given in footnote b of Table 2. N = 135.

a denotes the actual 1956 gross income and e the theoretical observation or predicted 1956 gross income.[25]

When the test was performed for the predicted logarithms of gross income and the actual logarithms of gross income for 1956, a chi-square value of 4.56 was obtained for the transcendental equation. A similar test using the parameters from the Cobb-Douglas function yielded a value of 4.37. These small values of chi-square indicate how closely the predicted agreed with the actual. With only one sample, one cannot attach a probability statement to the credulity of the hypothesis. However, the probability of finding the predicted deviating from the actual in repeated sampling from the same population is extremely small. By knowing this, it may be concluded that the hypothesis is true or it has been confirmed until further notice.[26]

SUMMARY OF MAJOR RESULTS

The results given in Tables 2 and 3 are of particular interest to researchers and to those who want to know the productivity of inputs. First, notice the contrasts between the two functions. While the Cobb-Douglas shows constant returns to scale throughout the range of inputs, the transcendental gives a similar result only at the mean level of inputs. In addition the estimates of marginal productivities are different; however, considering the size of their standard errors these differences may not be significant from a statistical standpoint. From the standpoint of predicting farmers' action in respect to the combining of inputs in least cost combinations, it remains to be seen which function provides the most accurate predictions.

Second, notice for the transcendental equation that only the standard errors of the marginal productivities are meaningful, i.e., it takes both coefficients to show the contribution of the variable. Thus, a test of significance on either regression coefficient would provide no information.

Third, notice that for the Cobb-Douglas the standard errors of the mean marginal productivity for inputs X_2, X_3, and X_4 are higher than for the transcendental function. Although the mean marginal productivity for these three inputs are higher for the Cobb-Douglas, the testing of predictions in the second phase of this study will provide a more powerful criterion upon which to judge the two equations.

Another important result from the standpoint of research methodology is the adjustment of the gross income for differences in weather. In this study gross income was first predicted without considering weather as a variable. The same

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[25]George W. Snedecor, Statistical Methods, (Ames: Iowa State College Press, 1946), pp. 16-18.

[26]Comparing actual 1956 gross income with the antilogarithmic value of the above predicted gross income will give exactly the same chi-square value.

equations as given above were fitted by least-squares regression to the unadjusted data. The chi-square test was used as it was above using the uncorrected 1952 coefficients and 1956 inputs to predict 1956 gross income. The chi-square value was 7.02 when comparing predicted with observed. This compared to a value of 4.37 when the corrected 1956 coefficients were used to predict 1956 income. Thus, the unexplained residuals were cut almost in half by considering the weather variable; this gives strong support to the adjustment procedure used, and credit is due those who developed the concept and measurement of drought days as an indicator of the weather variable.

To demonstrate the effect of omitting the weather variable on the estimation of marginal productivity of inputs, Table 4 presents the marginal productivities before and after adjusting for weather at the arithmetic mean levels of the four input categories given in Table 5.

TABLE 4. - Marginal productivities for arithmetic mean levels of input categories for a sample of Western Kentucky farms, 1952 (income adjusted and unadjusted for weather).[a]

Marginal Productivity of Input Category	X_1	X_2	X_3	X_4
Before weather corrected	22.98	3.30	.27	.71
After weather corrected	33.73	3.91	1.13	1.05

[a]The given marginal productivities for the transcendental equation are derived by the formula:

$$\frac{\delta Y}{\delta X_i} = Y\left(\frac{a_i}{X_i} + b_i\right)$$

In this study the problem of adjusting for weather was particularly crucial since predictions over time were being made.[27] In addition predictions of changes in combinations of inputs will be made, and when these are based upon the level of unadjusted marginal productivities serious errors are likely to result. For example, since the uncorrected marginal productivity of X_3 is extremely low compared to its one dollar cost, one might predict that farmers would use less in 1956 than they did in 1952. In fact as can be seen in Table 5 they used more of this input. A more accurate prediction could have been made from the corrected marginal productivity. The importance of adjusting output for weather in productivity studies cannot be overemphasized.

[27]If weather had not been accounted for it would have been impossible to determine whether or not the function had changed.

TABLE 5. - Geometric and arithmetic mean levels of gross income and input categories for a sample of West Kentucky farms 1952 and 1956.[a]

	Y ($)	X_1 (index)									X_2 (Days)	X_3 ($)	X_4 ($)
			V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8			
		Index	(Acres)	(Acres)	($)	($)	(Sq. ft.)	($)	($)	($)			
1952													
Arithmetic	4,494	100	102.3	3.23	280.76	193.84	1,185	73.86	194.37	68.75	343	435.29	1,289.93
Geometric	3,568	85.1	-	-	-	-	-	-	-	-	302	218.25	922.25
Weather adjusted 1952													
Arithmetic	6,366	X	X	X	X	X	X	X	X	X	X	X	X
Geometric	5,284												
1956													
Arithmetic	6,681	100	99.2	3.61	296.85	219.60	1,113	66.08	222.27	66.79	341	573.58	1,171.79
Geometric	4,931	79.3	-	-	-	-	-	-	-	-	291	303.50	807.00

[a] Y = Gross income
X_1 = Acres of land and associated inputs
V_1 = operating acres of land
V_2 = tobacco acreage
V_3 = depreciation on tractors, trucks, cornpickers, graindrills, and combines
V_4 = machinery repairs
V_5 = area of tobacco barn
V_6 = fence depreciation
V_7 = fuel expense
V_8 = crop seed expense

X_2 = days of labor used
X_3 = forage-livestock investment
X_4 = purchased inputs

APPENDIX A

METHODS OF EVALUATING INPUT AND OUTPUT ITEMS

Gross Income

Sales of tobacco, grains, seed, livestock, livestock products, other income from crops, and services performed were figures taken from each farmer -- actual dollars sold.

Home-used products such as garden produce, milk, and eggs are determined as follows: quantity figures were taken from each farmer and price figures were taken from Agricultural Statistics,[1] except in some cases where actual dollars sold were taken from each farmer as above.

Home-produced feed and seed inventory quantity figures were given by each farmer. When the closing inventory value was greater than the beginning inventory value, the difference was entered as a positive figure and vice versa. Price figures for these items were taken from Agricultural Statistics.

Closing inventory values of feeder livestock were determined from quantity figures obtained from each farmer and prices from Agricultural Statistics.

Forage Machinery, Forage Consuming Livestock, and Others

Machinery including hay balers, forage harvesters, trailers and wagons, ensilage cutters, feed grinders, grass seeders, mowing machines, milk coolers, and milkers was entered as depreciation charges. Quantity figures were determined from each farm and prices from Agricultural Statistics and farm equipment guides. Straight line depreciation rates were used with various life expectancies for the items.

Major farm buildings including dairy barns, silos, cattle barns, and cattle sheds were also entered as depreciation charges. Straight line depreciation rates were used with a 50-year life expectancy. Dollar figures were determined for each farm, and price indices from Agricultural Statistics were used to adjust the prices paid for building material.

Dairy cows, dairy bulls, beef cows and bulls, ewes and dams were entered as depreciation charges with a straight-line depreciation rate being used. The difference between the beginning inventory value and salvage value was depreciated

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[1] U. S. Department of Agriculture, Agricultural Statistics, 1952 and 1956.

over a six-year period. Values were determined from farmer estimates and from Agricultural Statistics. Heifers and calves, hogs and chickens were evaluated by subtracting value of the beginning inventory from the closing inventory value. The difference was entered as the observation. Quantity figures and estimated of value were obtained from each farmer; prices were obtained from Agricultural Statistics. Beginning inventory values of feeder calves, pigs, and lambs were also entered in this category.

Expenses such as forage seed, custom hay baling, field chopping, and veterinarian fees were entered as actual dollars spent, obtained from each farmer.

Purchased Inputs

Each item in this category was determined from actual dollars spent for each item for each farm. Farm share of expenses such as electricity, telephone, and automobile was estimated.

Other Items

Some items of the X_1 category were in physical terms and others in dollars, e.g., both depreciation charges on investments and expenses. Machinery depreciation rates were straight-line; prices of these items were obtained from farm equipment guides and Agricultural Statistics. Fuel, crop seed expense, and machinery repairs figures were obtained from farmers in dollars. Operating acres and tobacco barn area were specified in quantity figures, obtained from each farmer.

Labor was specified in days used per farm including hired labor, family labor, and the manager's labor.

Prices for a number of items of the variables specified in dollars are shown in the subsequent tables of this appendix.

TABLE 1. - Prices for gross income items[a] for 1952 and 1956 and the 1956 index.

Item	1952 Price	1956 Price	Index[b]
Tobacco (per cwt)			
Dark Air-Cured	$29.90	$31.38	105
Dark Fire-Cured	35.39	33.31	94
Burley	50.33	63.50	126
Field Corn (per bushel)	1.51	1.30	86
Popcorn (per cwt)	4.44	2.72	61
Wheat	2.09	1.97	94
Oats	.79	.69	87
Seeds (per cwt)			
Lespedeza	17.90	10.20	57
Alfalfa	32.70	30.90	94
Orchard Grass	16.60	21.30	128
Red Top	37.70	30.90	94
Timothy	13.60	15.50	114
Fescue (Red)	31.30	42.30	88
Fescue (Tall)	25.40	13.10	52
Ladino Clover	91.50	35.60	39
Red Clover	31.50	33.00	105
Soybeans (per bushel)	2.73	2.17	80
Strawberries (per lb for processing)	.159	.146	92
Hay (per ton - all grades)	33.20	20.50	62
Alfalfa	26.30	21.00	80
Mixed Clover & Timothy	22.80	20.00	88
Custom Work:			
Cornpicking (per acre)	5.00	5.50	110
Combining (per acre)	5.00	6.00	120
Hay Baling (per bale)	.15	.15	100
Rented Pasture	.....	.....	180
Feeder Livestock (all grades)			
Cattle (per cwt)	32.38	22.00	68
Lambs (per cwt)	26.90	22.04	83
Broilers (per cwt)	.28	.20	68
Milk Cows (per head)	243.00	153.00	63
Veal Calves (per cwt)	34.42	23.62	62
Beef Cattle (per cwt)	24.30	14.90	61
Hogs (including pigs per cwt)	26.00	16.30	63
Chickens (all-per lb)	.26	.19	72
Milk (all grades - per cwt)	5.50	4.16	76
Milk (grade A - per cwt)	5.85	4.58	78
Eggs (per doz)	.41	.39	95
Wool (per lb)	.53	.45	85
Garden (index)	.....	.....	99

[a]Some items for which prices were seldom used and are not included in the table.

[b]The index is computed by making the 1956 price an index or percentage of the 1952 price.

TABLE 2. - Prices for selected items of the X_1 input category for 1952 and 1956.

Item	1952 Price	1956 Price	Index
Tractors			
One-Plow Size	$1,520	$1,460	96
Two-Plow Size	1,980	2,100	106
Three-Plow Size	2,740	2,770	106
Combines			
12-Foot Self-Propelled	5,290	5,820	110
Auxiliary Engine 12-Foot Cut	3,380	3,640	108
Power-Take-Off, 5-6 Foot Cut	1,460	1,580	108
Corn Pickers (2-row)	1,500	1,840	123
Grain Drills			
Tractors, 12-Tube	476	510	107
Tractors, 20-Tube	662	722	109
Trucks			
2-Ton	2,330	2,660	114
Pickup	1,640	1,740	106
Building Materials (index)	.....	.....	107
Fuel - Motor Supplies (index)	.....	.....	107
Fences - Barbed Wire	8.68	9.31	107
Fences - Woven Wire	19.70	20.20	103
Machinery Repairs (index)	.....	.....	105
Crop Seed (per bu)			
Hybrid Corn (per bu)	11.00	10.90	99
Oats (per bu)	1.86	1.41	76
Barley (per bu)	2.39	1.91	80
Wheat (per bu)	3.08	2.88	94
Rye (per bu)	3.10	2.42	78
Soybeans (per bu)	4.23	3.73	88

TABLE 3. - Prices for selected items of the X_3 input category for 1952 and 1956

Item	1952 Price	1956 Price	Index
Hay Balers (automatic twine tie)			
Pick-up, Power-Take-Up	$2,110	$2,312	110
Pick-up, Motor Drive	2,665	2,852	107
Mowing Machines (tractor)			
5 Foot	227	230	101
7 Foot	309	338	109
Wagons, Less Tires	157	171	109
Milking Machines (single unit)	112	121	108
All Other Machinery (general index)	.....	.....	107
Building Supplies (buildings)	.....	.....	107
Others[a]			

[a]See Table 1 of this appendix for prices of livestock, forage seed, and custom rates.

TABLE 4. - Prices for selected items of the X_4 input category for 1952 and 1956.

Item	1952 Price	1956 Price	Index
Purchased Feed (per cwt)	.....	.....	82
Soybean Oil Meal	$ 5.55	$ 4.01	72
Laying Mash	5.27	4.48	85
Poultry Ration	4.21	3.55	84
Dairy Feed (18% protein)	4.51	3.66	81
Stock Salt	1.42	1.52	107
Purchased Feeders (per cwt)			
Cattle	26.30	16.00	61
Pigs	19.40	15.60	80
Lambs	24.00	18.40	77
Fertilizer (per ton)			
4-12-8	45.00	46.00	102
33 1/3% NH_4NO_3	85.00	80.00	94
47% Phosphate	60.00	67.00	112
5-10-10	32.40	50.80	97
Potash (Muriate)	60.00	56.00	93
Lime (per ton)	4.00	4.00	100
Electricity (per kilowatt hour)	.0290	.0271	93
Telephone (per month)	3.06	3.50	114
Baby Chicks	15.60	15.00	96
Spray Materials:			
Lead Arsenate (per lb)	.40	.36	90
Paris Green	.62	.68	110
Sawdust	15.00	15.00	100

APPENDIX B

TABLE 1. - Coefficients of determination (r^2) among input categories for 1952 and 1956.

Input Item	X_1		X_2		X_3		X_4	
	1952	1956	1952	1956	1952	1956	1952	1956
X_1	1.00	1.00	.30	.31	.11	.21	.41	.56
X_2	....	....	....	....	.03	.04	.20	.37
X_3	....	....	....	....	....	....	.11	.22
X_4	....	....	....	....	....	....	1.00	1.00

TABLE 2. - Coefficients of determination (r^2) among input categories in the Jensen-Sundquist study for 1952[a].

Input Item[b]	X_1	X_2	X_3	X_4
X_1	1.00	.24	.53	.48
X_2	....	1.00	.19	.29
X_3	....	....	1.00	.46
X_4	....	....	....	1.00

[a]H. R. Jensen and W. B. Sundquist, Resource Productivity and Income for a Sample of West Kentucky Farms, Bulletin 630, (Lexington: Kentucky Agricultural Experiment Station, June, 1954).

[b]The inputs were:

X_1 denoted acres of land

X_2 denoted months of labor

X_3 denoted crop services

X_4 denoted livestock services